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BY

M. A. STEPHENS

TECHNICAL REPORT NO. 449
DECEMBER 2, 1991

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Tests of fit for the Cauchy distribution based on the empirical distribution function

by

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Abstract

Points are given for testing goodness-of-fit to the Cauchy distribution, with unknown location and/or scale parameters. The tests are based on the empirical distribution function, and the asymptotic points round off work begun by Darling (1955) on the asymptotic theory of test statistics. Monte Carlo points are given for finite n and some discussion of power is included.

Key words: Goodness-of-fit Tests.

1. INTRODUCTION.

In a pioneering paper, Darling (1955) discussed the asymptotic theory of the empirical process and of certain goodness-of-fit statistics based on this process, when parameters must be estimated from the sample used in testing fit. The estimated parameters were location and scale parameters, and the theory was illustrated by a test for the Cauchy distribution. The statistics discussed were the Cramer-von Mises W^2 and the Anderson-Darling A^2, statistics based on the empirical distribution function (EDF) of the given sample.

In this article we develop the tests for the Cauchy distribution, when either or both of the location and sclae parameters are estimated by efficient estimators given below. The tests are set out in Section 2. Asymptotic percentage points are given for W^2 and A^2, and also for the EDF statistic U^2 introduced by Watson (1961); they involve calculating weights in sums of weighted chi-square variables. This is done by techniques drawn from Darling (1955) and the details are given in Section 3. For finite samples, points for the three statistics have been found from Monte Carlo samples. Points for the well-known Kolmogorov statistic D, and for the related V were found at the same time, and a table for Case 3 is given for reference; the asymptotic theory used for the Cramer-von Mises statistics cannot be applied to these statistics. D is usually not as powerful as W^2 or A^2, although V is sometimes competitive with U^2. A brief discussion of alternative statistics, and power, is given in Section 4.

2. TESTS FOR THE CAUCHY DISTRIBUTION.

Suppose a given random sample is $X_1, X_2, \ldots, X_n$, with order statistics $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$. The test discussed is a test of

H_0: the X-sample comes from the distribution

$$F(x;\alpha,\beta) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1}\left(\frac{x-\alpha}{\beta}\right), \ -\infty < x < \infty \tag{1}$$

with density function

$$f(x;\alpha,\beta) = \frac{1}{\pi\{1 + (x-\alpha)/\beta\}^2}, \ -\infty < x < \infty . \tag{2}$$

We can distinguish 4 cases, following Stephens (1974):

Case 0: parameters α and β in (1) are both known.

Case 1: parameter α is not known, β is known.

Case 2: parameter α is known, β is not known.

Case 3: parameters α, β are both unknown.

In Cases 1, 2 and 3 estimates of α, β are obtained from the formulas

$\hat{\alpha} = \Sigma_i \, a_i \, X_{(i)}$ and $\hat{\beta} = \Sigma_i \, d_i \, X_{(i)}$, where

$$c_i = \frac{\sin[4\pi\{i/(n+1) - 0.5\}]}{n \tan[\pi\{i/(n+1) - 0.5\}]} \tag{3}$$

and

$$d_i = 8 \sin[\pi\{i/(n+1) - 0.5\}]\cos^3[\pi\{i/(n+1) - 0.5\}]/n . \tag{4}$$

Here, and in later formulas, sums run for i from 1 to n.

These are not maximum likelihood estimators (MLEs) but are asymptotically efficient, an important requisite for the asymptotic theory of Section 3 to be valid. These estimators are used because MLEs are known to be difficult to work with; the likelihood can have local maxima and it is sometimes difficult to decide on the global maximum. When the estimates are obtained, the test continues with the following steps:

(1) Calculate $z_{(i)} = F(X_{(i)};\alpha,\beta)$, replacing α and/or β by estimates where necessary;

(2) Calculate the three test statistics from

$$W^2 = \Sigma_i\{z_{(i)} - (2i-1)/(2n)\}^2 + 1/(12n) \tag{5}$$

$$U^2 = W^2 - n(\bar{z} - 0.5)^2 \text{ , where } \bar{z} = \Sigma_i z_{(i)}/n \tag{6}$$

$$A^2 = -n - n^{-1} \Sigma_i (2i-1)\{\log(z_{(i)}) + \log(1 - z_{(n+1-i)})\}. \tag{7}$$

Here log refers to natural logarithm.

(3) Refer the value of the statistic used to Table 1, for the appropriate Case: H_0 is rejected at significance level α if the test statistic exceeds the value given for the sample size n and the desired level α.

The present Table for Case 0 is a more accurate update of a previously published table, (Stephens, 1974, 1976) although the changes are trivial in practice. For other Cases, the distributions of W^2, U^2 and A^2 do not depend on the true α, β. The asymptotic points are calculated from the theory in the next section, and points for finite n are based on

Monte Carlo studies using 10,000 samples for each n. It can be seen that for this very heavy-tailed distribution, and with these estimators the points vary with n somewhat surprisingly; for other distributions (see, e.g., Stephens, 1974, 1977, 1979), and with MLES, they converge much more rapidly to the asymptotic points.

The statistics D and V are obtained from the $z_{(i)}$ by

$D^+ = \max_i\{(i/n) - z_{(i)}\}$; $D^- = \max_i\{z_{(i)} - (i-1)/n\}$;

$D = \max(D^+, D^-)$ and $V = D^+ + D^-$.

Monte Carlo points for $D\sqrt{n}$ and for $V\sqrt{n}$ are given in Table 2, based on the same 10,000 samples as for the statistics W^2, U^2 and A^2.

3. THEORY OF THE TESTS.

The asymptotic distribution of any one of the three statistics is that of

$$S = \Sigma_i \; u_i/\lambda_i \; , \qquad i = 1,2,\ldots \tag{8}$$

where u_i are independent χ_1^2 variables and λ_i are weights. The weights are found from the now-classical asymptotic theory of the empirical process of the z-values. Darling (1955) gave this theory for tests for absolutely continuous distributions and illustrated it for W^2 with the Cauchy distribution, although details of how the λ_i are calculated were omitted except for Case 2. We now complete the calculations, following the steps and notation given in Stephens (1976, 1977). The empirical process, for all cases, becomes asymptotically a Gaussian process $Z(s)$, with $E(Z(s)) = 0$, $Z(0) = Z(1) = 0$, and with the covariance $\rho(s,t) \equiv E(Z(s)Z(t))$ varying with the Case. In Case 0 $\rho(s,t) = \rho_0(s,t) = \min s,t - st$. For the other three cases $\rho(s,t)$ takes the following form:

Case 1: $\rho(s,t) = \rho_0(s,t) - \phi_1(s)\phi_1(t)$

Case 2: $\rho(s,t) = \rho_0(s,t) - \phi_2(s)\,\phi_2(t)$

Case 3: $\rho(s,t) = \rho_0(s,t) - \phi_1(s)\phi_1(t) - \phi_2(s)\phi_2(t)$ with

$$\phi_1(s) = -\sqrt{2}(\sin^2 \pi s)/\pi \quad \text{and} \quad \phi_2(s) = (\sin 2\pi u)/(\sqrt{2}\pi).$$

These results for Cases 1 and 2 were given by Darling: the simple result for Case 3 follows because the estimates of α and β are asymptotically independent and the Fisher information matrix is diagonal (see Stephens, 1976, 1977). For Cases 1 and 2 the weights λ_i are found as follows. First calculate

$$a_j = \int_0^1 \phi_1(s) \sin \pi js \, ds$$

and

$$b_j = \int_0^1 \phi_2(s) \sin \pi js \, ds \ , \quad j = 1,2,\ldots .$$

and define

$$S_a(\lambda) = 1 + \lambda \sum_{j=1}^{\infty} \frac{a_j^2}{1-\lambda/(\pi^2 j^2)} \ , \quad S_b(\lambda) = 1 + \lambda \sum_{j=1}^{\infty} \frac{b_j^2}{1-\lambda/(\pi^2 j^2)} \ .$$

Let $d_0(\lambda)$ be the Fredholm determinant associated with $\rho_0(s,t)$: $d_0(\lambda) = \prod_j(\lambda - \pi^2 j^2)$. For Case 1, the Fredholm determinant is $D_1(\lambda) = d_0(\lambda)\, S_a(\lambda)$ and for Case 2 it is $D_2(\lambda) = d_0(\lambda)\, S_b(\lambda)$. The weights for these Cases are found by solving $D_1(\lambda) = 0$ for Case 1 and $D_2(\lambda)$ for Case 2.

Case 1. It is easily shown that $a_j = 0$ for j even, and $a_j = 8/\{\pi^2 j(j^2 - 4)\}$ for j odd. Setting $D_1(\lambda) = 0$ gives a set of solutions $\lambda_i^* = \pi^2 j^2$, $j = 2,4,6,\ldots$; another set is found by solving $S_a(\lambda) = 0$ (the solutions $\lambda_j = \pi^2 j^2$ of $d_0(\lambda)$, for j odd, are not solutions of $D_1(\lambda) = 0$ because of cancellation with the denominators in $S_a(\lambda)$). To solve $S_a(\lambda) = 0$ it is best to write $\kappa = 1/\lambda$ and solve $S_a^*(\kappa) = 1 + \sum_{j=1}^{\infty} a_j^2/\{\kappa - 1/(\pi^2 j^2)\} = 0$; a solution κ_i exists in each interval $(1/(3^2\pi^2),\ 1/\pi^2)$, $(1/(5^2\pi^2),\ 1/(3^2\pi^2))$, etc. and these are easily found numerically.

Case 2. For Case 2, $b_j = 0$ except $b_2 = 1/2\pi$. The solutions of $D_2(\lambda) = 0$

are then $\lambda_j = \pi^2 j^2$ except for $j = 2$. This rather curious result is remarked on by Darling as losing a "degree of freedom" by the estimation of β.

Case 3. For Case 3, the Fredholm determinant becomes $D_3(\lambda) = d_0(\lambda)S_a(\lambda)S_b(\lambda)$, (Stephens, 1976) and setting $D_3(\lambda) = 0$ gives two sets of λ; the set λ^* of $S_a(\lambda) = 0$ already found as part of the solution for Case 1, and the set $\lambda_j^{**} = 1/\pi^2 j^2$, for all j except $j = 2$, found for Case 2.

Cumulants of asymptotic distributions. The cumulants of the distributions can be found by direct calculations The mean for Case j is $\mu_j = 1/6 - \int_0^1 \phi_j^2(s)ds$, $j = 1,2$; the values are $\mu_1 = 1/6 - 3/(4\pi^2) = 0.0907$ (note a misprint in Darling, 1955, Section 8A) and $\mu_2 = 1/6 - 1/(4\pi^2) = 0.1413$. For Case 3, $\mu_3 = 1/6 - 1/\pi^2 = 0.0653$. Other cumulants may be calculated as described in Stephens (1976). The values are given for reference in Table 3. They may be used to provide checks on the calculations of the λ_j, since they may also be calculated from the distributional form $S = \Sigma_i u_i/\lambda_i$. The r-th cumulant is $\kappa_2 = 2^{r-1}(r-1)!\ \Sigma_i\ 1/(\lambda_i)^r$; these converge sufficiently fast, for $r \geq 2$, to give excellent checks on the λ_i values by matching with the direct calculations.

When the λ_i were found, for the different cases, Imhof's (1961) method was used to give the percentage points for S. The points were checked, with excellent agreement, by fitting Pearson curves to the distribution, using the first four cumulants. The slight changes from earlier tables, for Case 0 points in Table 1 are due to replacing Pearson curve fits by points found from the Imhof method.

Statistic U^2. The asymptotic distribution of U^2 is that of $\int_0^1 Z_1^2(t)dt$ where $Z_1(t) = Z(t) - \int_0^1 Z(t)dt$. (Watson, 1961). The solutions for λ_i are somewhat more complicated in principle (see Stephens, 1976) but in fact, for the Cauchy distribution, they work out easily; details will be omitted. For Case 1, the weights are the set $\lambda_j^* = 4\pi^2 j^2$, $j = 1,2,\ldots$ and a second set λ_j^{**} which are identical to λ_j^* except that λ_1^* is omitted. For Case 2, the weights work out to be the same as those for Case 1, a surprising result which means that the asymptotic distribution of U^2 is the same in both Cases. This occurs also for the logistic distribution; see Stephens (1979). For Case 3 the weights are two sets of λ_j^{**}. The calculations for cumulants give the values in Table 3.

Statistic A^2. For A^2 the process $Q(t) = Z(t)/\omega(t)$ must be examined, where $\omega(t) = \{t(1-t)\}^{\frac{1}{2}}$. The details parallel those given in Stephens (1976) for the normal distribution. For Case 0, the weights, solutions of the corresponding Fredholm determinant $d_0(\lambda)$, are $\lambda_j = j(j+1)$, $j = 1,2,\ldots$. For Case 1, the a_j work out to be zero for j even and must be found numerically for j odd. The weights λ_j are then the set $\lambda_j^* = j(j+1)$, $j = 2,4,6,\ldots$ and a second set λ_j^{**} which are solutions of $S_a(\lambda) = 0$. For Case 2, $b_j = 0$, for j odd, and the weights are the set $\lambda_j^* = j(j+1)$, $j = 1,3,5,\ldots$, and the second set λ_j^{**}, solutions of $S_b(\lambda) = 0$. For Case 3 the weights are the two sets λ_j^{**} for Cases 1 and 2. The calculations for cumulants give the values in Table 3.

4. FINAL REMARKS.

There are not many tests available for testing fit to the Cauchy distribution. In this article we have given points for EDF tests, tests which are consistent and unbiased and which for other distributions, are very effective in terms of power.

Possible alternative tests might be made using the correlation coefficient of the $X_{(i)}$, against m_i, where m_i is the expected value of the ith order statistic of a sample of size n from (1), with $\alpha = 0$ and $\beta = 1$. The values of m_i are not easily obtained, and m_i might therefore be replaced by $H_i = F^{-1}(r;0,1)$ with $r = i/(n+1)$. H_i is a well known approximation for m_i for most distributions; the approximation is less good in the tails, and of course for the Cauchy distribution the tails will be important. However, H_i is easily calculated and tables based on the correlation coefficient between $X_{(i)}$ and H_i, called $R(X,H)$, have been given by Stephens (1986). The tables are for $Z(X,H) = n(1-R^2(X,H))$, and are given for complete and also for right-censored samples. Other possible approaches to testing fit include tests based on spacings and tests based on the empirical characteristic function. It is hoped to develop such tests for practical use, and to include them, with EDF and correlation statistics, in an extensive power study. Preliminary work suggests that EDF statistics are much better than correlation statistics, at least, with U^2 and V best overall.

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Table 1

Upper tail percentage points for W^2, U^2 and A^2.

Significance level α .

n	.25	.15	.10	.05	.025	.01
			Case 1. Statistic W^2			
5	.208	.382	.667	1.26	1.51	1.61
8	.227	.480	.870	1.68	2.30	2.55
10	.227	.460	.840	1.80	2.60	3.10
12	.220	.430	.770	1.76	2.85	3.65
15	.205	.372	.670	1.59	2.88	4.23
20	.189	.315	.520	1.25	2.65	4.80
25	.175	.275	.420	.870	2.10	4.70
30	.166	.250	.360	.710	1.60	4.10
40	.153	.220	.290	.510	1.50	3.05
50	.145	.200	.260	.400	.70	2.05
100	.130	.170	.210	.270	.35	.60
∞	.115	.146	.173	.216	.260	.319
			Case 2. Statistic W^2			
5	.199	.236	.261	.338	.437	.590
8	.211	.273	.321	.389	.463	.564
10	.212	.279	.332	.414	.501	.626
12	.212	.281	.337	.433	.525	.661
15	.206	.279	.339	.444	.537	.684
20	.199	.273	.333	.442	.547	.698
25	.194	.268	.328	.437	.551	.704
30	.189	.265	.326	.435	.553	.708
40	.185	.260	.323	.434	.555	.712
50	.183	.258	.321	.433	.557	.714
100	.179	.254	.319	.432	.559	.715
∞	.176	.250	.316	.431	.560	.714
			Case 3. Statistic W^2			
5	.167	.242	.305	.393	.445	.481
8	.192	.315	.441	.703	.940	1.13
10	.197	.331	.481	.833	1.201	1.571
12	.194	.329	.487	.896	1.391	1.901
15	.185	.317	.472	.904	1.54	2.33
20	.169	.281	.419	.835	1.63	2.96
25	.154	.253	.366	.726	1.47	3.08
30	.143	.225	.319	.615	1.25	2.90
40	.126	.195	.263	.460	.850	2.17
50	.117	.175	.235	.381	.642	1.56
60	.1097	.160	.211	.330	.508	1.07
100	.098	.135	.174	.2378	.331	.544
∞	.080	.108	.130	.170	.212	.270

CASE 1. STATISTIC U^2

5	.122	.173	.227	.315	.387	.407
8	.121	.185	.270	.470	.600	.650
10	.119	.180	.260	.500	.720	.800
12	.114	.172	.240	.505	.780	.960
15	.109	.158	.220	.480	.813	1.160
20	.100	.141	.190	.380	.780	1.340
25	.095	.128	.170	.280	.650	1.340
30	.090	.121	.150	.235	.480	1.230
40	.084	.110	.140	.195	.330	.970
50	.080	.104	.130	.170	.250	.600
100	.074	.095	.110	.145	.180	.250
∞	.071	.088	.105	.133	.163	.204

CASE 2. STATISTIC U^2

5	.120	.140	.156	.183	.202	.217
8	.122	.154	.177	.221	.280	.358
10	.119	.149	.175	.226	.296	.400
12	.115	.144	.169	.225	.294	.430
15	.109	.137	.161	.210	.276	.403
20	.101	.126	.148	.190	.247	.355
25	.095	.118	.137	.176	.220	.305
30	.091	.113	.131	.166	.202	.270
40	.086	.105	.123	.154	.187	.240
50	.082	.102	.117	.148	.180	.230
100	.076	.096	.111	.138	.169	.210
∞	.011	.088	.105	.133	.163	.204

CASE 3. STATISTIC U^2

5	.105	.133	.160	.202	.226	.243
8	.107	.151	.198	.293	.386	.466
10	.104	.150	.203	.324	.461	.597
12	.100	.144	.200	.339	.504	.712
15	.093	.132	.183	.330	.542	.844
20	.083	.116	.159	.295	.548	.974
25	.075	.101	.134	.242	.486	.999
30	.069	.091	.117	.202	.402	.940
40	.062	.079	.096	.147	.274	.697
50	.057	.070	.085	.121	.197	.505
60	.054	.066	.078	.104	.149	.344
100	.047	.057	.065	.080	.098	.154
∞	.047	.047	.052	.060	.070	.081

Case 1. Statistic A^2

5	1.19	2.22	3.83	8.00	12.75	17.980
8	1.33	2.62	4.7	10.0	17.4	25.0
10	1.34	2.52	4.5	10.6	18.2	29.0
12	1.31	2.42	4.1	9.9	18.8	32.0
15	1.30	2.15	3.5	8.2	17.2	31.2
20	1.17	1.86	2.8	6.5	14.4	27.5
25	1.12	1.68	2.3	4.7	10.8	23.0
30	1.08	1.55	2.1	3.8	8.2	20.0
40	1.02	1.38	1.8	2.9	5.2	15.5
50	.970	1.29	1.6	2.4	3.8	10
100	.890	1.16	1.4	1.8	2.2	3.5
∞	.834	1.02	1.219	1.519	1.812	2.212

Case 2. Statistic A^2

5	.974	1.131	1.239	1.59	2.08	2.84
8	1.085	1.360	1.560	1.88	2.18	2.55
10	1.110	1.414	1.653	2.04	2.38	2.89
12	1.117	1.443	1.710	2.14	2.55	3.15
15	1.117	1.449	1.728	2.22	2.65	3.31
20	1.101	1.444	1.728	2.24	2.73	3.44
25	1.083	1.432	1.727	2.25	2.77	3.50
30	1.064	1.422	1.724	2.25	2.80	3.53
40	1.051	1.41	1.723	2.26	2.82	3.56
50	1.045	1.405	1.722	2.27	2.83	3.59
100	1.038	1.40	1.718	2.28	2.86	3.64
∞	1.034	1.409	1.716	2.283	2.872	3.677

Case 3. Statistic A^2

5	.835	1.14	1.40	1.77	2.00	2.16
8	.992	1.52	2.06	3.20	4.27	5.24
10	1.04	1.63	2.27	3.77	5.58	7.50
12	1.04	1.65	2.33	4.14	6.43	9.51
15	1.02	1.61	2.28	4.25	7.20	11.50
20	.975	1.51	2.13	4.05	7.58	14.57
25	.914	1.40	1.94	3.57	6.91	14.96
30	.875	1.30	1.76	3.09	5.86	13.80
40	.812	1.16	1.53	2.48	4.23	10.20
50	.774	1.08	1.41	2.14	3.37	7.49
60	.743	1.02	1.30	1.92	2.76	5.32
100	.689	.927	1.14	1.52	2.05	3.30
∞	.615	.780	.949	1.225	1.52	1.90

Table 2

Upper tail percentage points for D and V , Case 3.

Statistic D

Significance level α .

n	.25	.15	.10	.05	.025	.01
10	1.05	1.22	1.42	1.75	2.06	2.37
12	1.00	1.22	1.42	1.83	2.22	2.62
20	.946	1.14	1.32	1.73	2.25	3.05
30	0.889	1.05	1.21	1.54	2.06	2.98
40	0.850	0.993	1.12	1.37	1.77	2.61
50	0.822	0.949	1.06	1.28	1.58	2.29
60	0.802	0.921	1.02	1.21	1.42	1.95
100	.755	.755	.925	1.07	1.23	1.49

Statistic V

n	.25	.15	.10	.05	.025	.01
10	1.30	1.48	1.65	1.96	2.27	2.57
12	1.31	1.48	1.65	2.01	2.39	2.79
20	1.24	1.39	1.53	1.89	2.36	3.15
30	1.18	1.30	1.42	1.69	2.20	3.09
40	1.15	1.25	1.34	1.53	1.91	2.74
50	1.12	1.21	1.30	1.46	1.72	2.40
60	1.10	1.19	1.26	1.40	1.47	2.10
100	1.06	1.14	1.20	1.30	1.41	1.64

Table 3

Cumulants of asymptotic distributions

		10μ	$\sigma^2 \times 10^2$	$\kappa_3 \times 10^3$	$\kappa_4 \times 10^4$
W^2	Case 0:	1.666	2.222	8.466	50.79
	Case 1:	.9068	.4052	.5099	.1015
	Case 2:	1.413	2.094	8.336	.5060
	Case 3:	.6585	.2769	.3799	.8187
U^2	Case 0:	.8333	.2777	.2645	.3968
	Cases 1,2:	.5800	.1495	.1345	.1992
	Case 3:	.327	.0211	4.51×10^{-3}	1.58×10^{-3}
		μ	σ^2	κ_3	κ_4
A^2	Case 0:	1	.5797	1.043	3.040
	Case 1:	.6638	.1872	.1579	.2137
	Case 2:	.8422	.5249	1.006	3.003
	Case 3:	.5060	.1324	.1211	.1768

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Goodness-of-fit tests

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Points are given for testing goodness-of-fit to the Cauchy distribution, with unknown location and/or scale parameters. The tests are based on the empirical distribution function, and the asymptotic points round off work begun by Darling (1955) on the asymptotic theory of test statistics. Monte Carlo points are given for finite n and some discussion of power is included.

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